# Modeling 2D Drift-Wave Turbulence with the Terry-Horton Equations

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## GOALS

- **Simulate 2D drift-waves with a linear driving term.**
- **Develop model to describe turbulence on both ITG and ETG (Ion/Electron Temperature Gradient) scales without solving for all wavenumbers in between.**

## BACKGROUND: 2D Drift-Wave Model

- We use the Terry-Horton equations, derived as follows ([1], [2], [3], [4]).
- Conservation of ion-guiding-center density gives:

$$\frac{\partial n_{gc}}{\partial t} + \boldsymbol{\nabla} \cdot [n_{gc}(\boldsymbol{V}_{\boldsymbol{E}\times\boldsymbol{B}} + u_{\parallel}\hat{\boldsymbol{z}})] = 0$$

- Here u// is the parallel ion flow, and ion-guiding-center density is related to the real-space ion density in the following way due to ion-polarization effects:

$$\mathrm{n}_i = n_{gc} + n_0 \frac{\rho_s^2 e}{T_e} \boldsymbol{\nabla}_{\perp}^{\mathbf{2}} \phi$$

- The drift velocity is given by:

$$\boldsymbol{V}_{\boldsymbol{E}\times\boldsymbol{B}} = \frac{c}{B}\hat{\boldsymbol{z}} \times \boldsymbol{\nabla}\phi$$

- Adding a linear drive, characterized by the delta_0 parameter, to the adiabatic (Boltzmann) electron distribution gives the following electron response:

$$\delta n_e = n_{eo}\left(1 - \delta_0 \rho_s \frac{\partial}{\partial y}\right)\frac{e\phi}{T_e}$$

- Quasi-neutrality gives the following relation between ion density and potential:

$$\tilde{n}_{gc} = \frac{n_0 e}{T_e}\left(1 - \rho_s^2 \boldsymbol{\nabla}_{\perp}{}^2 - \delta_0 \rho_s \frac{\partial}{\partial y}\right)\phi$$

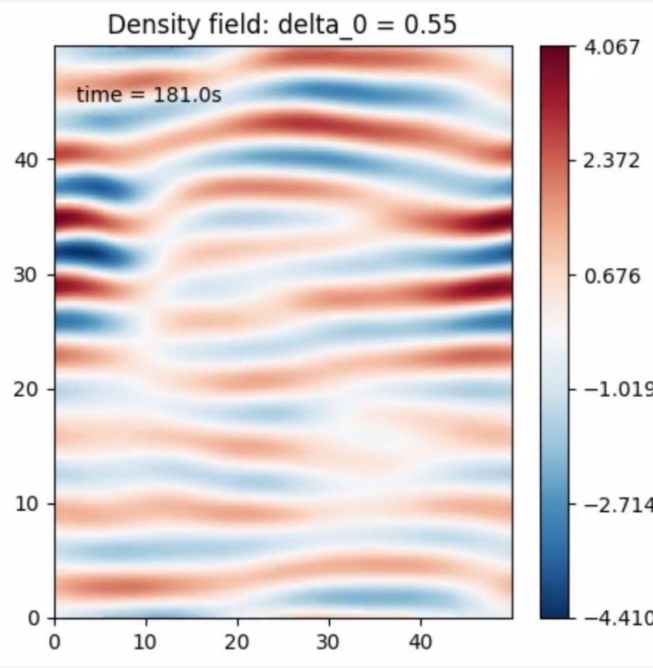
- We approximate the ion-guiding-center density as a linear function of x (the radial direction), having a gradient scale length L_n, and we consider the short-scale variations of ion density. We introduce the diamagnetic drift velocity V_d, and describe u// via a Landau damping model. Adding a dissipative term involving viscosity (mu) we arrive at the following:

$$\frac{\partial \tilde{n}_{gc}}{\partial t} + \boldsymbol{V}_{\boldsymbol{E}\times\boldsymbol{B}} \cdot \boldsymbol{\nabla}\tilde{n}_{gc} + n_0 V_d \frac{\partial}{\partial y}\left(\frac{e\phi}{T_e}\right)$$

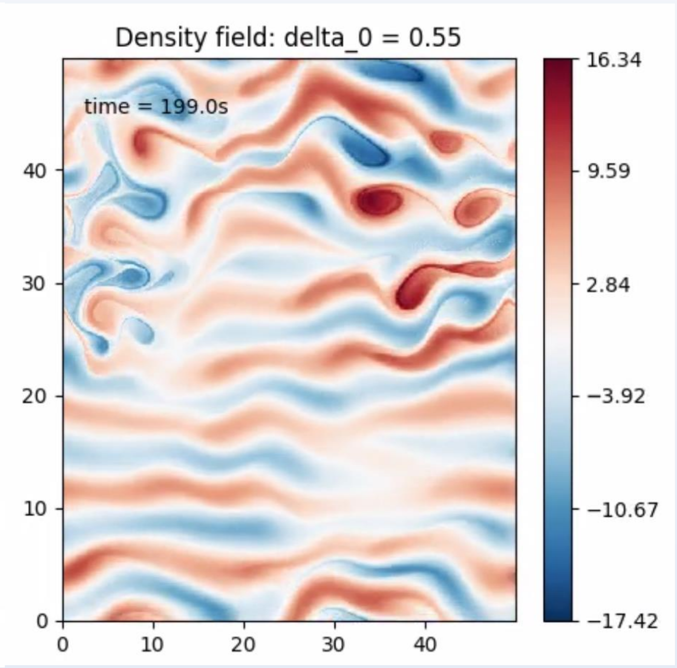
$$= -\alpha \frac{c_s}{L_n}\tilde{n}_{gc} + \mu \frac{c_s \rho_s^2}{L_n} \boldsymbol{\nabla}_{\perp}{}^2 \tilde{n}_{gc}$$

## TRANSITION TO TURBULENCE

- From random initial conditions, during the stage of linear instability, density waves form in the poloidal (y) direction, appearing as thin, long 'streamers' (shown right (a)).
- These waves grow in amplitude by a factor of $10^5$, according to the modes' linear growth rates (γ). They then break into turbulent vortices (shown right (b)), due to a Kelvin-Helmholtz secondary instability. ([5], [6])
- This is because the non-linear term in the equation only becomes significant as the density perturbations become large (the simulation shows the deviation of the density from the background value).
- The vortices are slightly radially elongated and cause particle transport in the radial (x) direction. This particle flux saturates in the nonlinear turbulence stage.




(a)




(b)

## MODIFIED ADIABATIC ELECTRON RESPONSE

- Following Dorland [5] and St-Onge [4], we modify the quasi-neutrality equation for the ky=0 modes such that electrons no longer respond to the zonal (ky=0) component of the potential, which prevents unphysical radial electron transport. For these modes we replace the quasi-neutrality equation with the following:

$$\tilde{n}_{gc} = \frac{n_0 e}{T_e}\left(-\rho_s^2 \boldsymbol{\nabla}_{\perp}{}^2 - \delta_0 \rho_s \frac{\partial}{\partial y}\right)\phi \qquad \text{For ky = 0 modes.}$$

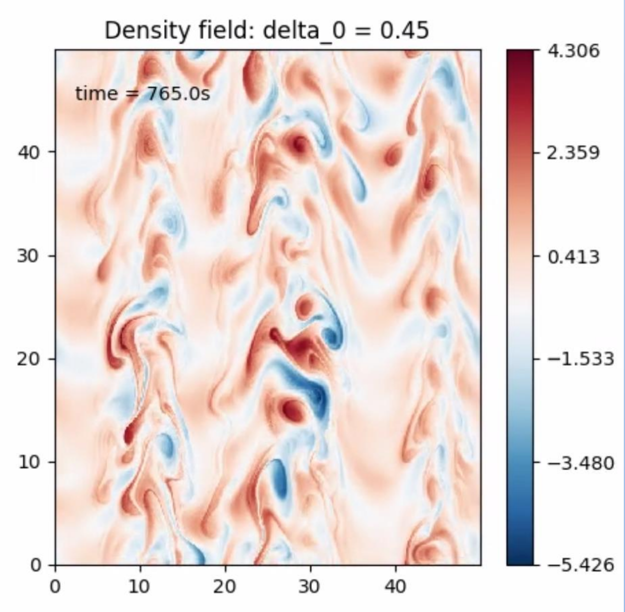
- This causes enhanced zonal flows and zonal flow shearing, and decreases the level of transport.
- These zonal flows are ExB flows in the poloidal (y) direction. For the ky=0 modes, phi is constant in the y direction, leaving:

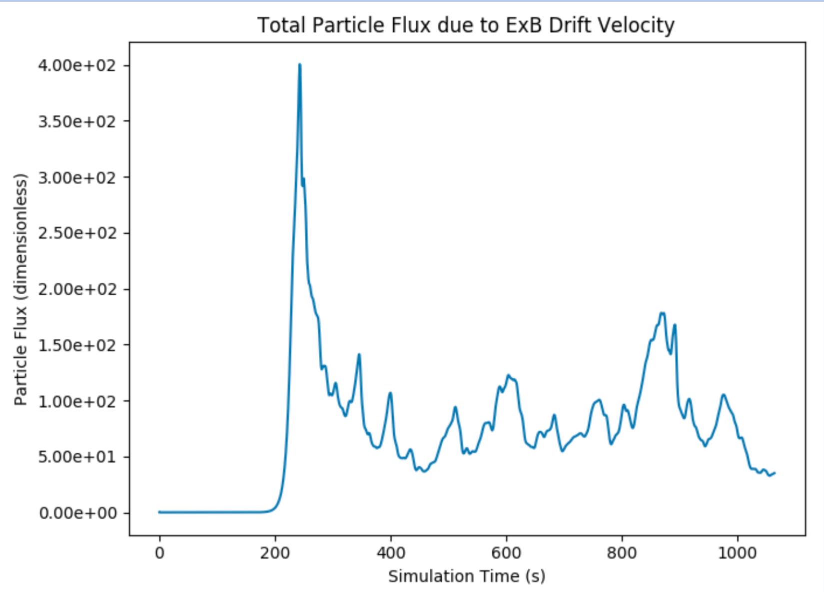
$$\boldsymbol{V}_{\boldsymbol{E}\times\boldsymbol{B}} = \frac{\partial \phi}{\partial x}\hat{\boldsymbol{y}} \qquad \text{For ky = 0 modes.}$$

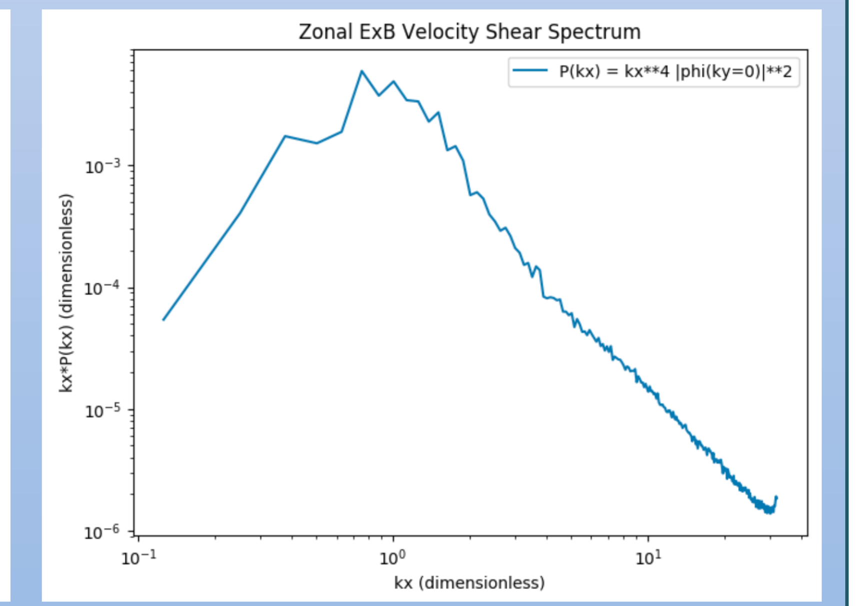
- A comparison of the modified (1) and unmodified (2) cases is shown below.
- Parameters: delta_0 (driving instability)=0.45*, alpha (models Landau damping)=0.035, mu (viscosity)=0.0001

**Density Snapshot during Turbulence** | **Particle Flux** | **Zonal Flow Shear Velocity Spectrum**

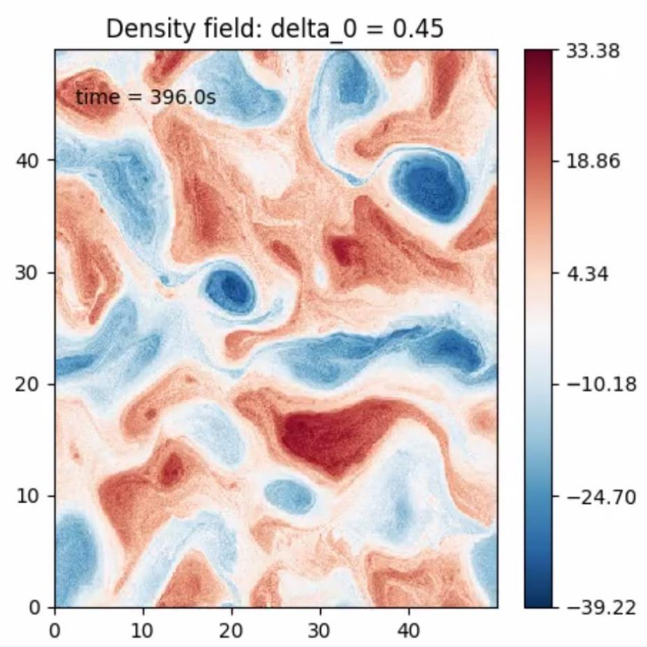
(1)

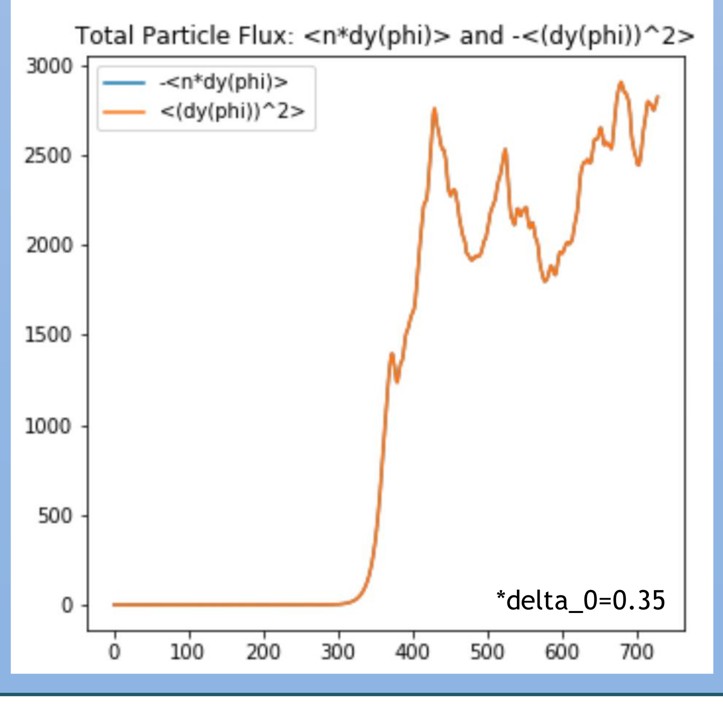


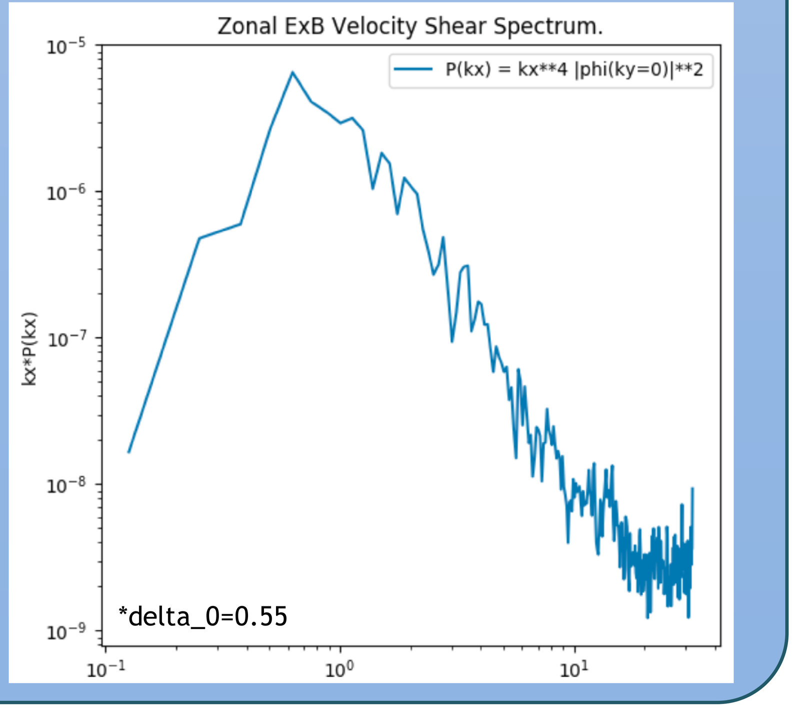




(2)



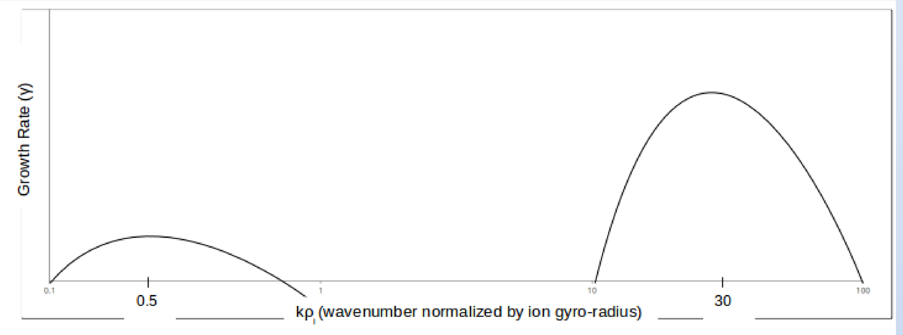




## SIMULATION METHOD

- The equations were implemented using a framework called Dedalus.
- "Dedalus solves differential equations using spectral methods. It is open-source, written in Python and MPI-parallelized." [7]
- Equations can be symbolically entered, initial conditions are easily set and the time step is calculated automatically at each iteration of the solver.

## FUTURE WORK

- It would be useful to replace the delta_0 parameter with a function giving a dispersion relation such that long and short scale turbulence is modeled.
- The model should give the desired behaviour at both ITG and ETG scales.
- This requires growth rates which peak at wavelengths comparable to the electron and ion gyro-radius respectively.




## CONCLUSIONS

- **Dedalus is an effective framework for simulating a plasma obeying the Terry-Horton equations.**
- **The modified adiabatic electron response introduces enhanced zonal flows and decreases radial transport.**

### REFERENCES

Papers:

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[3] Terry, P. and Horton, W. (1982). Stochasticity and the random phase approximation for three electron drift waves, The Physics of Fluids 25, 491.

[4] St-Onge, D.A. (2017), On non-local energy transfer via zonal flow in the Dimits shift. Princeton University, Princeton, NJ.

[5] Dorland, W.D. (1993), Gyrofluid models of plasma turbulence, Unpublished doctoral dissertation. Princeton University, Princeton, NJ.

[6] Rogers, B.N., Dorland, W. and Kotschenreuther, M. (2000), Generation and Stability of Zonal Flows in Ion-Temperature-Gradient Mode Turbulence, Physical Review Letters, Vol 85, 25.

[7] http://dedalus-project.org/

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